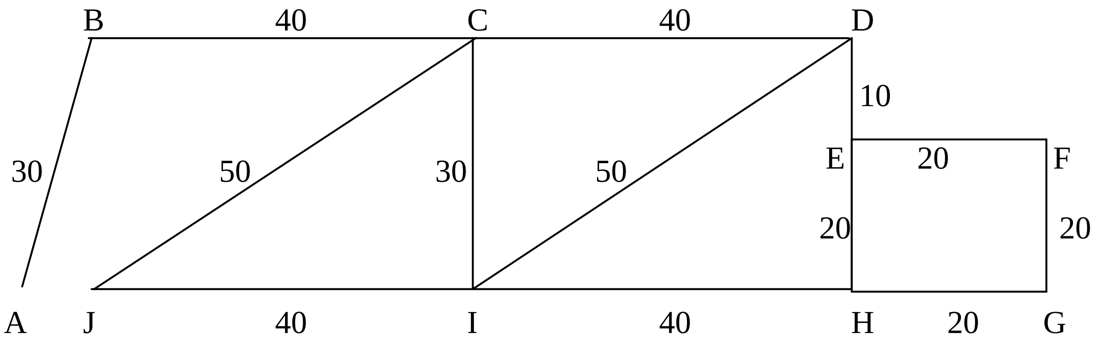
DECISION MATHEMATICS (C) UNIT 1 **TEST PAPER 9**

1. The final tableau of a Simplex calculation is

P	x	y	z	r	s	
1	2	0	0	4	7	9
0	3	1	0	5	-2	6
0	4	0	1	-1	3	4

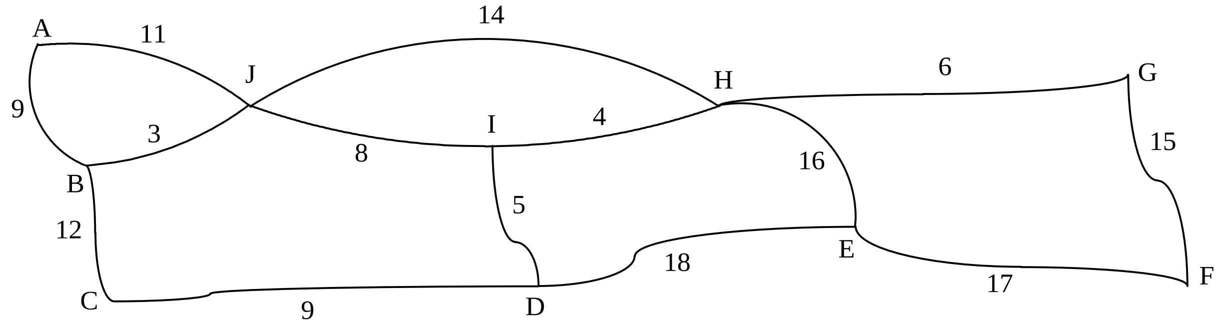
Write an expression for P in terms of the variables x, r and s, and hence explain why P has a maximum value of 9. State the values of y and z that generate this value of P. [5]

2. The figure shows the paths through an ornamental garden. A visitor wants to ensure that she walks along each path at least once.



(i) Use a suitable algorithm to find the minimum distance she must travel, starting and finishing at A. [5]
(ii) Write down a possible route of this minimum length. [2]
(iii) Explain why all networks have an even number of vertices of odd valency. [2]

3. (i) A tree has N nodes and A arcs. Write down a formula relating N to A. [2]
(ii) Use Kruskal's algorithm to find the minimum spanning tree for the network shown, clearly stating the order in which the arcs are selected. State its length. [4]



(iii) Sketch the minimum spanning tree, and verify that it satisfies the formula in part (i). [3]

4. A Sixth Former living in London wishes to spend a week, Monday to Friday, visiting each of his UCAS choices. The distances between each pair of universities are as follows :

	London	Aston	Nottingham	Sheffield	Bristol	Exeter
London		120	131	168	122	200
Aston	120		52	76	90	164
Nottingham	131	52		44	145	218
Sheffield	168	76	44		183	256
Bristol	122	90	145	183		84
Exeter	200	164	218	256	84	

(i) Use the Nearest Neighbour Algorithm to find an upper bound for the total length of his journey. [4]

(ii) Exeter then insists that he can only visit on the Wednesday of that week (whilst the others allow a free choice of visiting day). He decides to find the shortest path from London to Exeter, with two intermediate visits, and then to return to London using the shortest path back through the remaining two towns. Show that there are 24 such sequences that he could consider, and use the Nearest Neighbour procedure to find an upper bound for this cycle. [5]

(iii) Compare your answers to parts (i) and (ii). [1]

5. (i) Use the Shuttle Sort algorithm to sort the following numbers into ascending order.

23 16 7 24 18 9.

State the number of comparisons and interchanges made. [6]

(ii) Write down the maximum number of interchanges that might be required when Shutle sorting a list of

(a) 6 numbers, (b) n numbers. [5]

6. A salesman sells washing machines and dishwashers. He needs to sell at least five of each per 40-hour week to keep his job. Washing machines take longer to sell : 30 minutes, against 20 minutes for a dishwasher.
The company makes £80 profit per washing machine, and £65 per dishwasher, and the salesman is expected to earn £1000 profit per week for the company. For himself, he earns £15 for each washing machine sold, and £8 for each dishwasher.
Suppose the number of washing machines sold in one week is x, and the number of dishwashers is y.

(i) Show that $3x + 2y \leq 240$ and $16x + 13y \geq 200$, and state two other inequalities that are implied in the problem. [3]

(ii) Display the four inequalities on a graph and indicate the feasible region. [5]

(iii) Find the maximum amount he can earn in a week. [4]

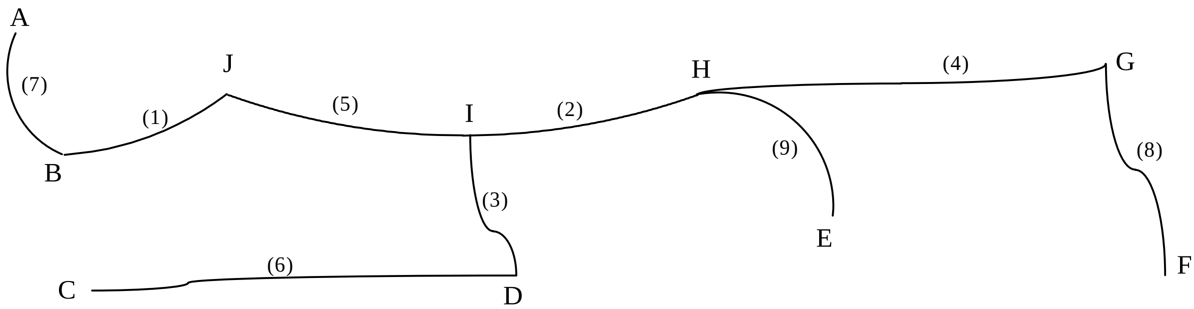
(iv) Find the minimum amount of time he must work each week, in order to fulfil his requirements. [4]

DECISION MATHS 1 (C) PAPER 9 : ANSWERS AND MARK SCHEME

1. $P = 9 - 2x - 4r - 7s$, so any increase of x, r or s above 0 will decrease P. B1 B1
   Thus P has a maximum value of 9, when $x = r = s = 0$, and $y = 6$, $z = 4$ M1 A1 A1 **5**

2. (i) Odd nodes are A, D, E and H. M1
   Possible pairings AD + EH = 130, AH + ED = 150 and AE + DH = 150 B1
   so repeat AD, EH Total distance = 410 + 130 = 540 M1 A1 A1
   (ii) e.g. A B C D E F G H E H I D C I J C B A M1 A1
   (iii) Each arc has two ends, so total number of "arc-ends" is even;
   even nodes give even contribution to this total, so the odd nodes' contribution must also be even. An odd number of odd nodes would give an odd contribution, so there must be an even number of odd nodes. B2 **9**

3. (i) $N = A + 1$ B2
   (ii) Use of algorithm, in correct order; length = 75 M1 M1 A1 A1



   (iii) Minimum spanning tree drawn M1 A1
   $N = 10$ and $A = 9$, so $N = A + 1$ B1 **9**

4. (i) L – A – N – S – B – E – L Total length = 683 M1 A1 M1 A1
   (ii) 4 options for Monday x 3 on Tuesday = 12; coming home, 2 options on M1
   Thursday x 1 on Friday = 2. Total number of possibilities = 12 x 2 = 24 A1 A1
   L – A – N – E – B – S – L Total length = 825 M1 A1
   (iii) The additional constraint in (ii) generally means that a less favourable result is achieved, as happens in this case B1 **10**

5. (i)

16	23	7	24	18	9	
7	16	23	24	18	9	M1 A1
7	16	18	23	24	9	
7	9	16	18	23	24	M1 A1

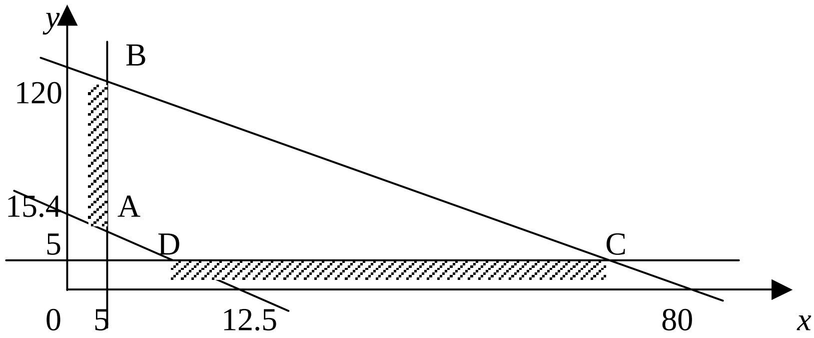
   12 comparisons and 8 interchanges A1 A1
   (ii) (a) 15 comparisons and interchanges M1 A1
   (b) $n(n-1)/2$ comparisons and interchanges M1 A1 A1 **11**

6. (i) Time : $30x + 20y \le 2400$ i.e. $3x + 2y \le 240$ B1

Profit : $80x + 65y \ge 1000$ i.e. $16x + 13y \ge 200$ B1

Also $x \ge 5$, $y \ge 5$ B1

(ii) Graphs :



B1 B1 B1

B1 B1

(iii) Maximum earnings at vertices B (5,112 ½) or C (76 $^2/_3$, 5) B1

Need to consider integer points: M1

at (5, 112), earnings £971, at (6, 111) £978, at (76, 5) £1180 and at (76, 6) £1188, so highest earnings when sells 76 washing machines and 6 dishwashers M1 A1

(iv) Least time at vertices A (5, 9 $^3/_{13}$) and D (8 $^7/_{16}$, 5) B1

Integer points: (5, 10) 350 minutes, (6, 9) 360 minutes, (9, 5) 370 minutes and (8, 6) 360 minutes. So he can work just 350 minutes and meet the requirements M1 M1 A1

16